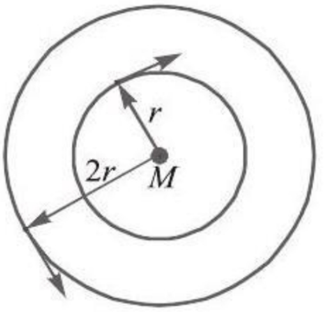
Gravitation

1. The energy required to take a satellite to a height ' h ' above Earth surface (radius of Earth $= 6.4 \times 10^{3}$ km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h (in km) for which E_1 and E_2 are equal, is:
2. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed (in m/s) that meteorite should have at O is: (Take Gravitational constant $G = 66 \times 10^{-11} \mathrm{Nm}^2\ \mathrm{kg}^{-2}$)
3. Two satellites, A and B, have masses m and 2 m respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius 2 R around the earth. The ratio of their kinetic energies, T_A/T_B, is $\frac{1}{x}$. Find the value of x.
4. The value of acceleration due to gravity at Earth's surface is $9.8\ \mathrm{ms}^{-2}$. The altitude (in metre) above its surface at which the acceleration due to gravity decreases to $4.9\ \mathrm{ms}^{-2}$, is close to : (Radius of earth $= 6.4 \times 10^{6}$ m)
5. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet? [Given: Mass of Planet $= 8 \times 10^{22}$ kg, Radius of planet $= 2 \times 10^{6}$ m, Gravitational constant $G = 6.67 \times 10^{-11} \mathrm{Nm}^2/\mathrm{kg}^2$]
6. The eccentricity of the earth's orbit is 0.0167. The ratio of its maximum speed in its orbit to its minimum speed is
7. A rocket is fired vertically from the surface of mars with a speed of 2 km/s. If 20% of its initial energy is lost due to martian atmosphere resistance, how far (in km) will the rocket go from the surface of mars before returning to it? Mass of mars $= 6.4 \times 10^{23}$ kg, radius of mars $= 3395$ km, $G = 6.67 \times 10^{-11} \mathrm{Nm}^2/\mathrm{kg}^2$.
8. Mass M is divided into two parts xM and $(1-x)M$. For a given separation, the value of x for which the gravitational attraction between the two pieces becomes maximum is
9. If the distance between the earth and the Sun were half its present value, the number of days in a year would have been
10. If the Earth has no rotational motion, the weight of a person on the equator is W. Determine the speed (in rad/s) with which the earth would have to rotate about its axis so that the person at the equator will weigh $\frac{3}{4}$ W. Radius of the Earth is 6400 km and $g = 10\ \mathrm{m/s}^2$.
11. Radius of moon is $1/4$ times that of earth and mass is $1/81$ times that of earth. The point at which gravitational field due to earth becomes equal and opposite to that of moon, is xR from centre of earth. Find the value of x. (Distance between centres of earth and moon is 60 R, where R is radius of earth)
12. The height (in km) of the orbit above the surface of the earth in which a satellite, if placed, will appear stationary is
13. An artificial satellite moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth's surface, will be at a height km.
14. The radius of the earth is reduced by 4%. The mass of the earth remains unchanged. The escape velocity increases by K%. Find the value of K.
15. Two satellites of masses m and $2m$ are revolving around a planet of mass M with different speeds in orbits of radii r and $2r$ respectively. The ratio of minimum and maximum forces on the planet due to satellites is $\frac{1}{x}$.



Find the value of x.

SOLUTIONS

1. **(3200)** K.E. of satellite is zero at earth surface and at height h from energy conservation

$U_{surface} + E_1 = U_h$

$-\frac{GM_e m}{R_e} + E_1 = -\frac{GM_e m}{(Re+h)}$

$\Rightarrow E_1 = GM_e m\left(\frac{1}{R_e} - \frac{1}{R_e+h}\right) \Rightarrow E_1 = \frac{GM_e m}{(R_e+h)} \times \frac{h}{R_e}$

Gravitational attraction

$F_G = ma_c = \frac{mv^2}{(R_e+h)} = \frac{GM_e m}{(R_e+h)^2}$

$mv^2 = \frac{GM_e m}{(R_e+h)}$

$E_2 = \frac{mv^2}{2} = \frac{GM_e m}{2(R_e+h)}$

$E_1 = E_2$

Clearly, $\frac{h}{R_e} = \frac{1}{2} \Rightarrow h = \frac{R_e}{2} = 3200 km$

2. **(2.8×10^5)** Let M is mass of star m is mass of meteorite

By energy convervation between 0 and ∞.

$-\frac{GMm}{r} + \frac{-GMm}{r} + \frac{1}{2}mV_{esc}^2 = 0 + 0$

$\therefore v = \sqrt{\frac{4GM}{r}} = \sqrt{\frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{10^{11}}}$

$\simeq 2.8 \times 10^5 m/s$

3. **(1)** Orbital, velocity, $v = \sqrt{\frac{GM}{r}}$

Kinetic energy of satellite A,

$T_A = \frac{1}{2} m_A V_A^2$

Kinetic energy of satellite B,

$T_B = \frac{1}{2} m_B V_B^2$

$\Rightarrow \frac{T_A}{T_B} = \frac{m \times \frac{GM}{R}}{2m \times \frac{GM}{2R}} = 1$

4. **(2.6×10^6)** Given

Acceleration due to gravity at a height h from earth's surface is

$g_h = g\left(1+\frac{h}{R_e}\right)^{-2} \Rightarrow 4.9 = 9.8\left(1+\frac{h}{R_e}\right)^{-2}$

$\frac{1}{\sqrt{2}} = \left(1-\frac{h}{R_e}\right)$ [as $h <<< R_e$]

$h = R_e(\sqrt{2}-1)$

$h = 6400 \times 0.414$ km

$h = 2.6 \times 10^6$ m

5. **(11)** Time period of revolution of satellite,

$$T = \frac{2\pi r}{v}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$\therefore T = 2\pi r\sqrt{\frac{r}{GM}} = 2\pi\sqrt{\frac{r^3}{GM}}$$

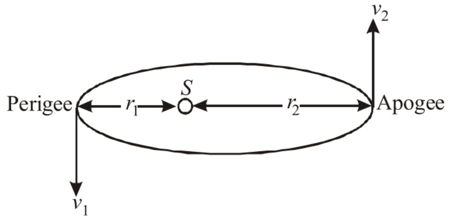
Substituting the values, we get

$$T = 2\pi\sqrt{\frac{(202)^3 \times 10^{12}}{6.67\times10^{-11}\times 8\times10^{22}}} \text{ sec}$$

T = 7812.2 s

$T \simeq 2.17$ hr $\Rightarrow$ 11 revolutions.

6. **(1.034)**



According to the law of conservation of angular momentum, $mv_1r_1 = mv_2r_2$

$$\frac{v_1}{v_2} = \frac{r_2}{r_1} = \frac{a(1+e)}{a(1-e)},$$

where e is eccentricity of the earth's orbit

$$= \frac{(1+0.0167)}{(1-0.0167)} = 1.034$$

7. **(1655)** If h is the height attained, then

$$0.8\times\left[\frac{1}{2}mv^2 - \frac{GMm}{R}\right] = -\frac{GMm}{R+h}$$

After substituting the given values, we get

$\therefore$ h = 1655 km

8. **(0.5)** $F \propto x\text{M}\times(1-x)\text{M} = x\text{M}^2(1-x)$

For maximum force, $\frac{dF}{dx} = 0$

$$\Rightarrow \frac{dF}{dx} = \text{M}^2 - 2x\text{M}^2 = 0 \Rightarrow x = 1/2$$

9. **(129)** By Kepler's law, $T^2 \propto a^3$

$$\Rightarrow \frac{T_1^2}{a_1^3} = \frac{T_2^2}{a_2^3}$$

$$\Rightarrow T_2^2 = \left(\frac{a_2}{a_1}\right)^3 \times T_1 = \left(\frac{\frac{1}{2}a_1}{a_1}\right)^3 \times T_1$$

$$= \frac{1}{8}\times T_1 = \frac{1}{8} \text{ years } (\because T_1 = 1 \text{ year})$$

$$\therefore T_2 = \frac{1}{\sqrt{8}} \text{ year } = \frac{1}{\sqrt{8}} \times 365 \text{ days } = 129 \text{ days}$$

10. **(0.6×10^{-3})** We know, $g' = g - \omega^2 R \cos^2\theta$

$$\frac{3g}{4} = g - \omega^2 R \Rightarrow \omega^2 R = \frac{g}{4}$$

Given, $g' = \frac{3}{4}g$

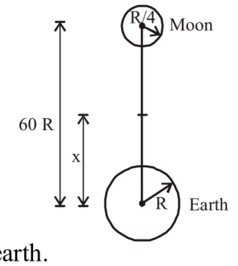
$$\omega = \sqrt{\frac{g}{4R}} = \sqrt{\frac{10}{4 \times 6400 \times 10^3}} = 0.6 \times 10^{-3} \text{ rad/s}$$

11. **(54)** $E_{earth} = E_{moon}$

$$\Rightarrow \frac{GM}{x^2} = \frac{GM/81}{(60R - x)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{9(60R - x)}$$

$\Rightarrow x = 54\text{ R}$ from centre of earth.



12. **(36000)**

13. **(6400)** $v_o = \frac{1}{2}v_e \Rightarrow \sqrt{\frac{gR^2}{R+h}} = \frac{\sqrt{2gR}}{2}$

$\therefore \ h = R = 6400\,\text{km}$

14. **(2)** Escape velocity $= v = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow v^2 = \frac{2GM}{R} \quad \text{.......(i)}$$

$v^2 = (2GM)R^{-1}$

Differentiating both sides, we get,

$$2v\frac{dv}{dR} = -\frac{2GM}{R^2} \Rightarrow v\frac{dv}{dR} = \frac{GM}{R^2} \quad \text{......(ii)}$$

Dividing (ii) by (i), $\frac{1}{v}\frac{dv}{dR} = -\frac{1}{2R}$

$$\Rightarrow \left|\frac{dv}{v}\right| \times 100 = \frac{1}{2} \times 4\% = 2\%$$

$\therefore$ If the radius decreases by 4%, escape velocity will increase by 2%.

15. **(3)** $F_{min} = \frac{GMm}{r^2} - \frac{GM(2m)}{(2r)^2} = \frac{GMm}{2r^2}$

and $F_{max} = \frac{GMm}{r^2} + \frac{GM(2m)}{(2r)^2} = \frac{3}{2}\frac{GMm}{r^2}$

$\therefore \ \frac{F_{min}}{F_{max}} = \frac{1}{3}$.